MATHEMATICS 3A/3B

Section Two:
Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student Number: In figures

In words

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
				Total	100

Instructions to candidates

1.   The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2014*. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer Booklet.

3.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5.   **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.   It is recommended that you **do not use pencil**, except in diagrams.

7.   The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **(100 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 **(5 marks)**

(a) Adam made the conjecture that the function $f(x) = x^2 - 3x + 7$ generated prime numbers for $1 \leq x \leq 9$, where x is an integer.

Was Adam's conjecture true? If not, state a value of x for which the conjecture is false.

(2 marks)

(b) Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of six for all positive integer values of n.

(3 marks)

Question 9 **(16 marks)**

A biologist was conducting trials of a new fertiliser. She obtained ten almost identical plants and applied different amounts of fertiliser to test its effectiveness. The heights of the plants were measured after two months. The results are shown in the table below.

Plant	1	2	3	4	5	6	7	8	9	10
Fertiliser (f) (grams)	8	9	11	12	13	15	17	20	21	23
Height (h) (centimetres)	24	26	29	28	32	33	35	35	34	33

(a) Determine the correlation coefficient r_{fh}. (1 mark)

(b) Describe, in words, the relationship between f and h. (2 marks)

(c) Determine the regression line of h on f. (1 mark)

(d) The difference between the actual plant height and the predicted plant height using the regression line for each different amount of fertiliser is calculated. This difference is called the residual.

   (i) Calculate the residuals for plants 9 and 10 and complete the table below. The first eight residuals are recorded in the table correct to two decimal places. (2 marks)

Plant	1	2	3	4	5	6	7	8	9	10
Residual	−2.47	−1.11	0.60	−1.04	2.32	2.04	2.75	0.83		

(ii) The residuals for the first eight plants have been plotted below. Complete the residual plot using the results for plants 9 and 10. (2 marks)
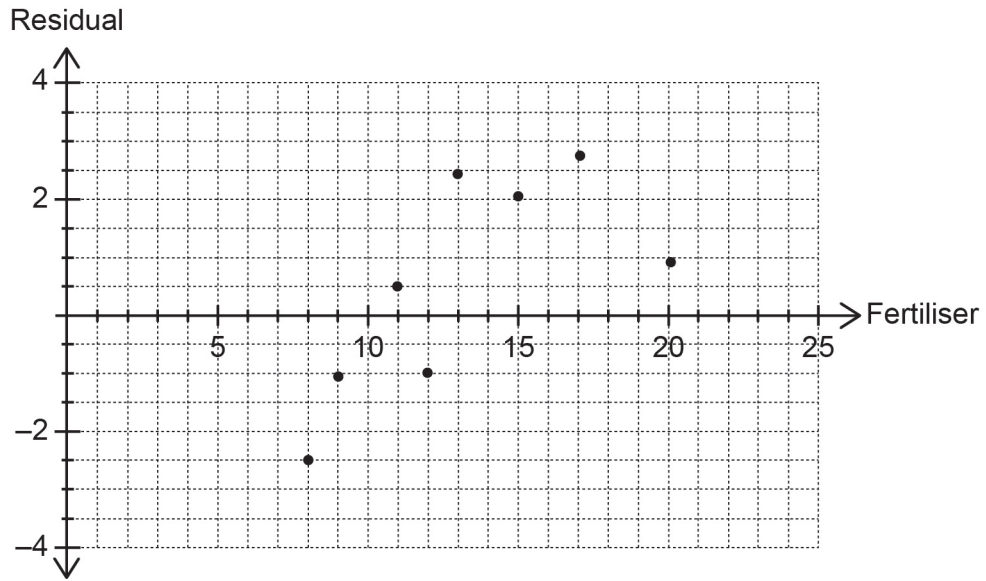
(iii) Comment on the information the residual plot revealed to the biologist. (2 marks)

(iv) State the coordinates $(\overline{f}, \overline{h})$, determine the corresponding residual for this point and plot it on the residual plot in part (ii). (3 marks)

(e) (i) Use the regression line to predict the height of a plant after two months when 22 grams of fertiliser are applied. (1 mark)

(ii) Comment on the reliability of the prediction in part (e) (i). (2 marks)

See next page

Question 10 **(8 marks)**

On 2 March, Michael is assigned to read a particular novel for his Literature class in preparation for a test on 22 March. On the first night, he reads 80 pages before falling asleep. On the next night, he reads 60 pages and on the third night, 45 pages. He continues to read 25% fewer pages each consecutive night.

(a) The situation can be represented by a recursive rule in the form $a_{n+1} = pa_n$, where $a_1 = q$.

Determine the values of p and q. (2 marks)

(b) How many pages, to the nearest whole number, will Michael read on the fifth night? (2 marks)

(c) How long will it take Michael to read at least 300 pages? (1 mark)

(d) If the book has 320 pages, will Michael complete the book by the test date? Justify your answer. (3 marks)

Question 11 **(6 marks)**

Consider the following data written in ascending order and consisting of seven integer values with the middle three values missing. The mean is 11.14 (correct to two decimal places) and the median is 11.

$$3, 4, \underline{\quad}, \underline{\quad}, \underline{\quad}, 17, 26$$

(a) If the smallest two scores from the data are both increased by one and the largest two scores are both reduced by one, state the effect on

(i) the standard deviation. (1 mark)

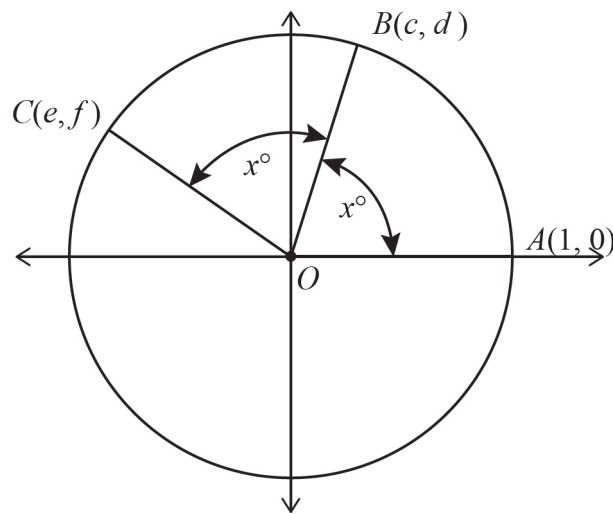(ii) the mean. (1 mark)

(iii) the median. (1 mark)

(b) Determine an eighth data point which, when included with the original data would produce the boxplots shown below.

(i) (1 mark)

2   8   26

(ii) (2 marks)

3   10.5   26

See next page
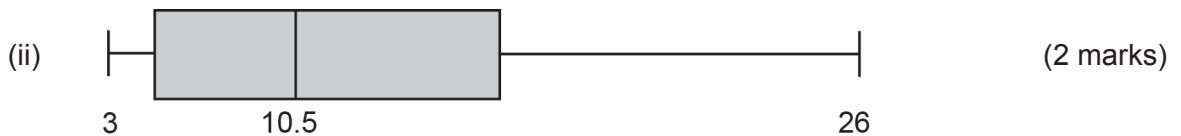
Question 12 **(7 marks)**

The unit circle below, with centre O at the origin, has the points $A(1, 0)$, $B(c, d)$ and $C(e, f)$ on the circumference and $\angle COB = \angle BOA = x°$, $45° < x° < 90°$.

Given Pythagoras's Theorem: 'In a right triangle PQR where p, q are the short sides and r is the hypotenuse, $r^2 = p^2 + q^2$,' use the pronumerals c, d, e and f to:

(a) state a relationship between c and d not involving x. (2 marks)

(b) determine an equation for $\sin x°$. (1 mark)

(c) evaluate $\cos 2x°$. (2 marks)

(d) determine the area of triangle OBC. (2 marks)

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Question 13 **(4 marks)**

A farmer wishes to fence a triangular paddock ABC. He first needs to calculate the perimeter of the paddock. He determines that point B is 45.8 m from point A and on a bearing of $055°$ and point C is 68.44 m from point A and on a bearing of $318°$.

(a) Draw a diagram to represent this information. (2 marks)

(b) Calculate the perimeter of the paddock. (2 marks)

Question 14 **(12 marks)**

The table below shows the cost of gas for a Perth household. Accounts are sent at the end of each time period. Accounts for 2010 are unavailable, but some statistics are provided.

Time (t)	Months/Year	Cost ($)	Four-point centred moving averages (m)	Residual
1	November 2009–January 2010	D	-	-
2	February 2010–April 2010	-	-	-
3	May 2010–July 2010	-	-	-
4	August 2010–October 2010	A	-	-
5	November 2010–January 2011	121.52	138.84	−17.32
6	February 2011–April 2011	140.14	139.07	1.08
7	May 2011–July 2011	153.26	140.13	C
8	August 2011–October 2011	143.11	141.73	1.38
9	November 2011–January 2012	126.47	144.68	−18.21
10	February 2012–April 2012	148.02	148.07	−0.05
11	May 2012–July 2012	169.00	151.15	17.85
12	August 2012–October 2012	154.45	152.33	2.12
13	November 2012–January 2013	139.78	152.58	−12.80
14	February 2013–April 2013	144.17	154.30	−10.13
15	May 2013–July 2013	174.80	155.32	19.48
16	August 2013–October 2013	162.43	155.80	6.63
17	November 2013–January 2014	139.96	156.73	−16.77
18	February 2014–April 2014	147.88	157.77	−9.89
19	May 2014–July 2014	178.50	-	-
20	August 2014–October 2014	167.05	-	-

(a) What is the time period of the household gas accounts? (1 mark)

(b) Calculate the missing entries marked by A and C. (2 marks)

(c)   Calculate the seasonal component for the November–January period.   (2 marks)

(d)   The regression equation for the time (t) against the four-point centred moving average, m, is $m = 1.534t + 131.72$. Predict the cost for the November 2014–January 2015 account.   (3 marks)

(e)   Comment on the reliability of your prediction from part (d).   (2 marks)

(f)   The seasonally adjusted figure for November 2009–January 2010 is $133.79. Determine the value of the missing entry marked by D.   (2 marks)

Question 15   **(12 marks)**

(a)   Given $f(x) = 7 - 3x$ with domain $\left\{ -2,\ 1,\ \dfrac{1}{3},\ 4 \right\}$, determine the range of $f(x)$.   (2 marks)

(b)   (i)   The function $g(x) = \sqrt{2(x+1)}$ can be formed by transforming $k(x) = \sqrt{x}$.
               Determine the transformations required to obtain $g(x)$ from $k(x)$ and state the
               transformations in the correct order.   (2 marks)

       (ii)   Determine the domain of $g(x)$.   (2 marks)

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(c)   (i)   Determine the equation of the function $h(x)$ shown on the axes below.   (3 marks)

$h(x)$

(2, 2)
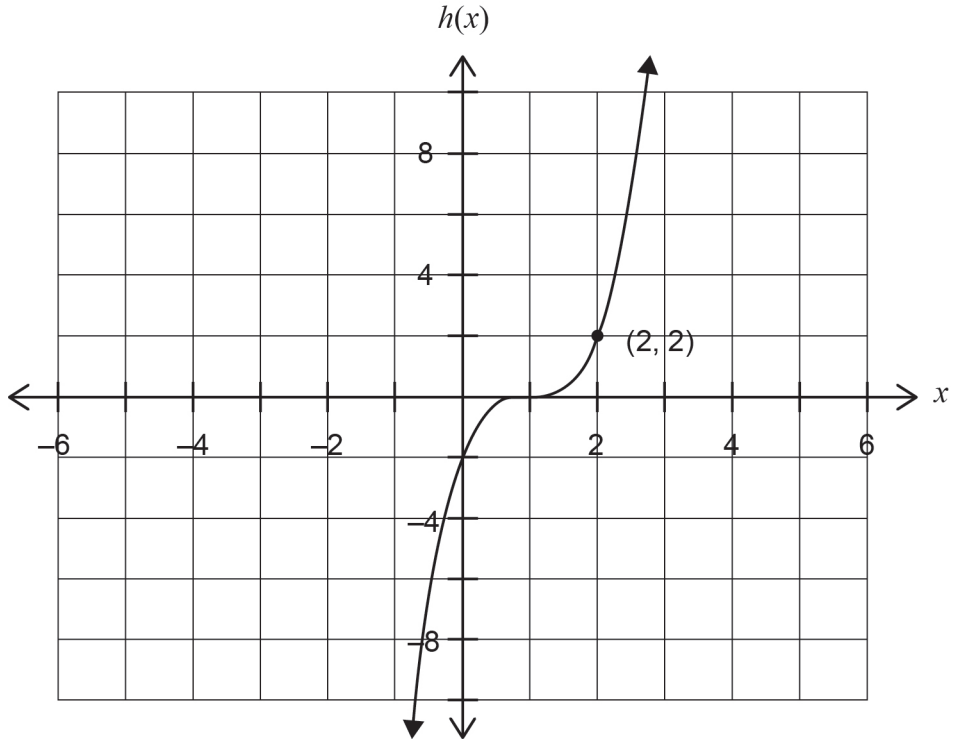
    (ii)   Use the axes above to sketch the function $y = h(-x) + 2$.   (3 marks)

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Question 16 **(9 marks)**

The cubic function $y = x^3 - 12x + 2$ is graphed below.
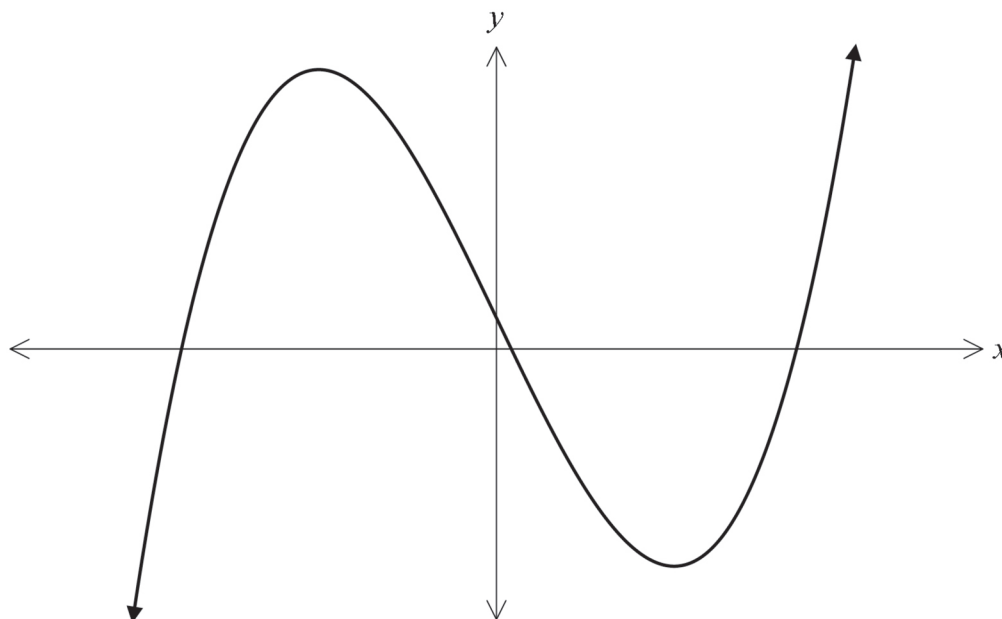
(a) By differentiating the function, determine the coordinates of the turning points. (4 marks)

(b) For what values of x is the function decreasing? (2 marks)

See next page

(c)   Determine the equation of the tangent to the curve at the point $(-1, 13)$.   (2 marks)

(d)   Verify that this tangent passes through the minimum turning point.   (1 mark)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 17 **(10 marks)**

A retailer determines that he can sell 60 headbands per week if he charges $10 per headband. The retailer estimates that for each 50 cent reduction in price he can sell 15 extra headbands per week.

(a) Complete the table below. (2 marks)

Number of 50 cent reductions	Price($)	Number of headbands sold
0	10	60
1	9.50	
2		90
3		
x	$10 - 0.5x$	

(b) Given that Revenue (R) = Sale Price (P) × Number Sold (N), and ignoring any other overhead costs, show that $R = 600 + 120x - 7.5x^2$. (2 marks)

(c) Using calculus techniques, calculate the amount that should be charged per headband to maximise the revenue. (3 marks)

(d) If the retailer initially purchased 200 headbands at $4 each, calculate his expected profit given that the price is set to maximise the revenue. (3 marks)

Question 18 **(3 marks)**

The heights of a small population of people living on a Pacific island were all measured for a regional census. It was found that 99.7% of the population were between 124 cm and 190 cm tall and 68% of the population were between 146 cm and 168 cm tall. If the heights are normally distributed, use this information to estimate the mean and standard deviation of the population heights.

Question 19 **(8 marks)**

A jeweller purchased cultured pearls for the production of necklaces. The diameters of the pearls were found to be normally distributed, with a mean of 1 cm and a standard deviation of 0.025 cm.

(a) What proportion of the pearls will have a diameter exceeding 1.07 cm? Give your answer correct to three significant figures. (2 marks)

(b) What is the probability that a pearl will have a diameter between 0.975 cm and 1.025 cm? (1 mark)

(c) Below what size will the diameter of 10% of the pearls fall? (2 marks)

Industry standards dictate that for the consistency of the necklace, pearls with a diameter of less than 0.93 cm or greater than 1.07 cm cannot be used.

(d) For every 1000 pearls, determine the number of pearls that will **not** be suitable for use in the necklaces. (3 marks)

End of questions

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